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On Increasing the Sensitivity for Certain Types of Experiments to Search for New Elementary Particles

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Abstract

We study the sensitivity of setting upper limits on the cross section for new particle production in high-energy collider experiments. Often the signature of a certain interaction process that creates new particles (a production mechanism) is the decay into a distinctive set of particles, the final state, that interact with the particle detector. Some models of new physics incorporating supersymmetry often predict that two or more correlated production mechanisms may be seen with the same final state. Each process can have a different probability of occurring (related to the cross section of a process) and a different efficiency of being detected. If an experiment yields a null result, one can set upper limits on the production cross section for each process individually and thus can potentially exclude the whole model if it predicts the cross section to be above the upper limit. Intuitively, one may want to try to set a limit for the individual process that gives the lowest cross section limit; however, we show that combining all of the production mechanisms into a single, effective production mechanism and setting a cross section limit on this combined production mechanism is the most sensitive way to search for new physics.

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1 Introduction

Particle physics theories often predict hypothetical new particles that can be produced in collisions in particle accelerators and detected by high energy experiments. In some cases there can be multiple interactions or processes for a model of new physics that produce some of the same final state particles, or particles that are observed in the detector. An excellent example is in supersymmetric models that predict new particles (for example, charginos and neutralinos) to be produced at the Fermilab Tevatron. In each case, the decay chains of the chargino events and of the neutralino events both result in the same signature of two photons plus two gravitinos, the latter being supersymmetric particles that can not be seen by the detector and whose footprint is missing energy [1]. Each process (in this case, the production of charginos or neutralinos) would have a different production cross section, which determines the likeliness of a process occurring (measured in picobarns, where 1 barn $= 10^{-24}$ cm^2), and an efficiency, which is the probability of detecting its occurrence in the detector. The typical experiment optimizes its detector settings to be sensitive to one, the other, or both the cross section and the efficiency.

If the experiment gives a null result in searching for a process, one may set an upper limit on the cross section for that process in a particular model [2]. In the case that a model provides two correlated production mechanisms (in our case neutralino or chargino production) that can be seen by the same final state (in our case two photons plus two gravitinos), one can put a limit on the total, or combined, production cross section as well as on each process individually. If the data result in an upper limit that is below the cross section predicted by a model, either for an individual process or for all processes together, that model is experimentally excluded.

If one process has a large cross section and a small efficiency, and the other has a small cross section and a large efficiency, one might wonder if it is more advantageous experimentally to put limits on the high cross section process, on the high efficiency process, or on both at the same time. In other words, should we optimize the experiment for the process that is more likely to occur but harder to detect or optimize for the one that is rare but easy to detect? In this paper we study this question.

2 Example Case

To answer this question, let us take the simple case of two correlated production mechanisms. We label them production mechanisms A and B, where process A has a cross section σ_A and an efficiency ε_A to be observed, and process B has a cross section σ_B and an efficiency ε_B. For an integrated luminosity L, equivalent to the number of collisions in our experimental data (measured in inverse picobarns), we expect the number of events produced by process A, N_A, to be given by

$$N_A \;=\; L \cdot \sigma_A \cdot \varepsilon_A \quad . \tag{1}$$

Since the same results hold for process B, the total number of events produced by both processes, N_T, is given by

$$N_T = N_A + N_B = L(\sigma_A \cdot \varepsilon_A + \sigma_B \cdot \varepsilon_B) = L \cdot \sigma_T \cdot \varepsilon_T \tag{2}$$

where we have defined

$$\sigma_T \equiv \sigma_A + \sigma_B \quad \text{and} \tag{3}$$

$$\varepsilon_T \equiv \frac{\sigma_A \cdot \varepsilon_A + \sigma_B \cdot \varepsilon_B}{\sigma_A + \sigma_B} \quad . \tag{4}$$

If we do a measurement and get a null result for process A, we can set a 95% confidence level upper limit cross section, σ_A^{95}, using

$$\sigma_A^{95} = \frac{N^{95}}{L \cdot \varepsilon_A} \tag{5}$$

where N^{95} is the 95% confidence level upper limit on the number of signal events allowed by the data, taking into account systematic errors [3]. In other words, we have a 95% probability that true cross section for process A could not be higher than σ_A^{95} because a cross section higher than σ_A^{95} would have produced more than N^{95} signal events 95% of the time. Note that N^{95} does not need a subscript to denote which process we are talking about because it is an experimental parameter that is the same for all processes. A theoretical model is excluded with 95% confidence level if it predicts a cross section that is above the upper limit calculated from the measurement. Thus, σ_A^{95} is an effective estimate of our experimental sensitivity to process A. This analysis can be repeated for process B if it produces the same final state. Combining the two processes allows one to set a total cross section limit:

$$\sigma_T^{95} = \frac{N^{95}}{L \cdot \varepsilon_T} \quad . \tag{6}$$

A single experiment can thus produce σ_A^{95}, σ_B^{95}, and σ_T^{95}, and our question, quantitatively, becomes which of the three is best to optimize when designing an experiment. At first glance one might want to optimize for the process that produces the lowest cross section limit, however this turns out to be a poor assumption. To illustrate why naively searching for the process which has the prospect of setting the lowest cross section limit is not the best method, we give a contrived example: $\sigma_A^{Theory} = 5.0$ pb, $\varepsilon_A = 5\%$, $\sigma_B^{Theory} = 1.0$ pb, and $\varepsilon_B = 20\%$. We use a value of 3.0 for N^{95} corresponding to zero observed events [2,3]. In other words, if we were expecting to see 3.0 events in the experiment, then the chance of observing exactly 0 events is 5%. For simplicity we assume a systematic error of 0% and use a luminosity of 10 pb^{-1} to calculate the cross section limits using Eqns. 5 and 6. The results are gathered in Table 1. As can be seen, neither process A nor process B is excluded; the upper limits for these processes are still above the theoretical prediction. Furthermore, σ_B^{95} is the lowest value, but it is *not* the most useful limit of the three. However, the combined limit allows us to exclude the model because it is below the total production cross section.

In other words, $\sigma_T^{95} \leq \sigma_T^{Theory}$ implies that the model is excluded by the experiment. The method of looking for a single process does not provide enough sensitivity to exclude the model with the given data, but combining both processes at the same time does provide increased sensitivity. In the next section we show generally that σ_T^{95} is always the best choice if the same analysis allows limits on both σ_A^{95} and σ_B^{95}.

Process	ε	σ^{Theory}(pb)	σ^{95}(pb)
A	5%	5.0	6.0
B	20%	1.0	1.5
Total	7.5%	6.0	4.0

Table 1. Example upper limits for a null experiment. We have assumed $N^{95} = 3.0$ and a luminosity of 10 pb^{-1} and used Eqns. 5 and 6. While process B gives the lowest cross section limit, neither process A nor process B is individually excluded, but the total combined result is excluded.

3 Proof

In the previous section we gave an example where σ_A^{95} and σ_B^{95} were not sensitive enough to exclude the model with the given data. However, σ_T^{95} was set from the same data, and it did have the sensitivity to exclude the model. We will now show that σ_A^{95} can never be more sensitive than σ_T^{95}. Our proof is to show that $\sigma_A^{95} \leq \sigma_A$ implies that $\sigma_T^{95} \leq \sigma_T$, where σ_A and σ_T are the values predicted by the model. In other words, if either A or B is excluded, then the total is always excluded.

For concreteness we begin by assuming that process A is excluded, or that

$$\sigma_A^{95} \quad \leq \quad \sigma_A \quad . \tag{7}$$

Using Eqn. 5, the formula for calculating the cross section limit, we get

$$\frac{N^{95}}{L \cdot \varepsilon_A} \quad \leq \quad \sigma_A \quad . \tag{8}$$

Next, we multiply the left side by the identity $(\frac{\varepsilon_T}{\varepsilon_T})$:

$$\frac{N^{95}}{L \cdot \varepsilon_A} \left(\frac{\varepsilon_T}{\varepsilon_T} \right) \quad \leq \quad \sigma_A \quad . \tag{9}$$

We then use the total efficiency in the denominator, and Eqn. 6, to rewrite the left side as

$$\sigma_T^{95} \left(\frac{\varepsilon_T}{\varepsilon_A} \right) \quad \leq \quad \sigma_A \quad . \tag{10}$$

Next, we multiply both sides by $(\frac{\varepsilon_A}{\varepsilon_T})$.

$$\sigma_T^{95} \quad \leq \quad \sigma_A \left(\frac{\varepsilon_A}{\varepsilon_T} \right) \tag{11}$$

Then we multiply the right side by $(\frac{\sigma_T}{\sigma_T})$ and rewrite it to get

$$\sigma_T^{95} \quad \leq \quad \sigma_A \left(\frac{\varepsilon_A}{\varepsilon_T} \right) \left(\frac{\sigma_T}{\sigma_T} \right) = \left(\frac{\sigma_A \cdot \varepsilon_A}{\sigma_T \cdot \varepsilon_T} \right) \sigma_T \quad . \tag{12}$$

Finally, we use Eqn. 2, where $\sigma_T \cdot \varepsilon_T \equiv \sigma_A \cdot \varepsilon_A + \sigma_B \cdot \varepsilon_B$, to get the following:

$$\sigma_T^{95} \quad \leq \quad \left(\frac{\sigma_A \cdot \varepsilon_A}{\sigma_A \cdot \varepsilon_A + \sigma_B \cdot \varepsilon_B} \right) \sigma_T \quad . \tag{13}$$

Since each term in Eqn. 13 is non-negative, we know that $\sigma_A \cdot \varepsilon_A \leq \sigma_A \cdot \varepsilon_A + \sigma_B \cdot \varepsilon_B$ and $0 \leq \frac{\sigma_A \cdot \varepsilon_A}{\sigma_A \cdot \varepsilon_A + \sigma_B \cdot \varepsilon_B} \leq 1$. It follows that if $\sigma_A^{95} \leq \sigma_A$ then Eqn. 14 must be true:

$$\sigma_T^{95} \quad \leq \quad \sigma_T \quad . \tag{14}$$

Therefore, if process A is excluded, then the total cross section limit also must be less than the theoretical total cross section value. Furthermore, this reasoning can be extended to more than two processes. If Eqn. 2 were replaced by

$$N_T \quad = \quad \sum_{i=1}^{n} N_i = L \sum_{i=1}^{n} (\sigma_i \cdot \varepsilon_i) \tag{15}$$

then Eqn. 13 can be rewritten as

$$\sigma_T^{95} \quad \leq \quad \left(\frac{\sigma_A \cdot \varepsilon_A}{\sum_{i=1}^{n} \sigma_i \cdot \varepsilon_i} \right) \sigma_T \tag{16}$$

and our result holds more generally.

We have just shown that the limit on the total cross section will always be more sensitive than the limit placed on any individual process. Thus, one should attempt to minimize the total cross section limit in a particle physics experiment of this type.

4 Conclusion

When a search for a model of new particle production in high-energy collider experiments returns a null result, one can place an upper limit on the cross section of any production mechanisms that could have been observed with the same final state. It is possible for one experiment to yield cross section limits on multiple processes separately and a combined limit on the total production cross section. We find that if a single search for one final state is performed then the sensitivity to new physics is maximized by optimizing the search to take into account all production mechanisms that can contribute to the final state. In other words, setting a limit on the total cross section can provide more sensitivity to a model than setting a limit on a single process; so, one should always optimize the experiment for the lowest total cross section limit for a given final state. This method has already been used by the Collider Detector at Fermilab (CDF) Collaboration in the search for gauge-mediated supersymmetry-breaking models with a light gravitino, providing some of the world's most sensitive limits to date [1].

References and Notes

[1] CDF Collaboration (D. Acosta *et al*), "Search for Anomalous Production of Diphoton Events with Missing Transverse Energy at CDF and Limits on Gauge-Mediated Supersymmetry-Breaking Models," *Physical Review D* **71**, 031104(R),(2005).

[2] W.-M. Yao *et al.*, "Review of Particle Physics," *Journal of Physics G* **33**, 1 (2006).

[3] According to Poisson statistics, N^{95} for a null result is determined by the confidence level ($N^{95} = -\ln(1 - .95) \approx 3.0$), and the confidence level is set to 95% by convention.