A SELF-CONSISTENT APPROACH TO THE LASER COOLING OF V-TYPE ATOMS

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ABSTRACT

The equations of motion for three energy level V-type atoms driven by two counter propagating laser fields are derived from Schrödinger's equation. These equations at steady state are reduced to a single integral equation for the ground state momentum distribution. A numerical method, based on a self-consistent approach, is developed to study the laser cooling of V-type atoms. A good agreement is reached between our numerical results and those from existing theory. [1]

[1] Y. Castin, H. Wallis, and J. Dalibard, J. Opt. Am., B 6, (1989), p. 2046.

INTRODUCTION

A single photon has a momentum $\hbar k$ and has an energy $\hbar\omega$, where $\hbar$ is Planck's constant divided by 2π and k is the wave number and ω is the frequency of the electromagnetic field of the photon. Due to momentum conservation, an atom of mass M and momentum p can gain or lose momentum of $\hbar k$ during the absorption or emission of a single photon. Consequently, the kinetic energy of the atom changes by:

$$\frac{(p \pm \hbar k)^2}{2M} - \frac{p^2}{2M} = \frac{\pm \hbar kp}{M} + E_r\,, \qquad (1)$$

where kp/M is the Doppler frequency shift and E_r is the atomic recoil energy:

$$E_r = \frac{(\hbar k)^2}{2M} = \hbar\omega_r\,, \qquad (2)$$

were ω_r is called the recoil frequency shift. These ideas form the basis for the laser cooling of atoms. [1]

Temperature is a measure of the average kinetic energy of the atoms in a system. The average kinetic energy is determined by the atomic momentum (or velocity) distribution. Atoms in thermal equilibrium follow the Maxwell-Boltzmann distribution; the broader the distribution the higher the temperature. The essential task of laser cooling is to narrow the momentum distribution profile by an exchange of momentum between atoms and photons. The simplest laser cooling scheme is when atoms of two energy levels are driven by two opposing laser fields of the same frequency. The Doppler effect causes the atoms to see the laser beam traveling in the direction opposite to their motion to have a higher frequency than that along their direction of motion. If the laser frequency is less than the atomic transition frequency and the motion of the atoms are slow enough, the frequency shift due to the Doppler effect will always bring the energy of the counter propagating laser closer to energy of the atomic transition than the copropagating one. As a result, the radiation force

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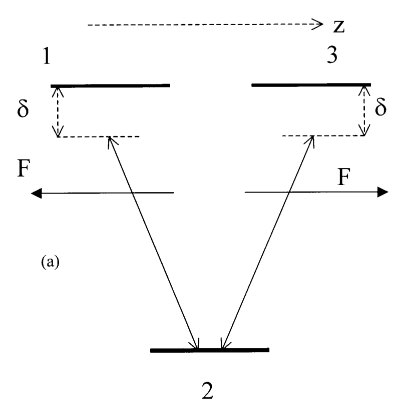

Figure 1

A schematic diagram of three-level 'V' type atoms driven by two counter propagating laser fields of amplitude F, and frequency detuning δ. The system is oriented along the z axis.

given to the atom by counter propagating laser is always larger than that given by the copropagating laser. This effect gives rise to a net force opposing the motion of the atoms, and results in the narrowing of the momentum distribution of the atoms and the reduction of the atomic temperature.

A V-type atom, the kind that is discussed in this paper, is a three energy level atom whose levels are arranged in a 'V' as shown in Figure 1. The transitions between the quantum states 1 -> 2 and 3 -> 2 are driven independently by two counter propagating laser fields of the same amplitude F, frequency ω and wave number k. We are interested in momentum distribution of the 'cooled' atoms, both in the broad distribution ($\Delta p >> \hbar k$) and narrow distribution ($\Delta p \approx \hbar k$), where Δp is the width of the atomic momentum distribution. In the case of the narrow momentum distribution, the deBroglie wavelength of the atom, $\lambda_D = h/p \approx h/\hbar k = 2\pi/k = \lambda$ is on the order of the wavelength of the laser field. In this case, the atoms can no longer be viewed as localized particles moving classically under the electromagnetic fields of the lasers.

A more accurate theoretical description requires the simultaneous quantization of both the internal and external (center-of-mass) degrees of freedom. In a theory developed for the 'V' system, the steady-state momentum distribution is obtained either by integrating the generalized Optical Bloch (GOB) equation over a long period of time or solving directly the steady-state coupled GOB equations. [2] The former is very time consuming, while the latter involves matrices of large dimension. In this work, we approach this problem in a self-consistent manner. In a self-consistent approach, we start from a guessed function and use the equation of interest to arrive at an improved solution in an iterative manner until the solution does not appear to change with further iteration. This method is easy to implement and quite efficient for atoms with narrow atomic lines.

EQUATIONS OF MOTION

The time dependent Schrödinger equation for this system is:

$$i\hbar\frac{\partial}{\partial t}|\psi\rangle = \left(\hat{H}_A + \frac{1}{2}\frac{\hat{p}^2}{M} + \hat{H}_{AL}\right)|\psi\rangle , \qquad (3)$$

where $\hat{H}_A$ represents the interaction among the particles inside the atom, $\frac{1}{2}\frac{\hat{p}^2}{M}$ is the kinetic energy of the center-of-mass and $\hat{H}_{AL}$ is the dipole interaction between the atom and the laser fields. The interaction of the atoms with the vacuum is not included in Equation 3 but will be introduced later phenomenologically. We choose to work in a space spanned by $|i,p\rangle$, defined as:

$$\hat{H}_A\,|i,p\rangle = \hbar\,\Omega_{i2}\,|i,p\rangle$$
$$\hat{p}\,|i,p\rangle = p\,|i,p\rangle \quad , \qquad (4)$$

where $\hbar\,\Omega_{i2}$ is the energy of the i^{th} level relative to the ground state energy and p is the eigenvalue of the momentum operator. The system is closed in the sense that:

$$\int_{-\infty}^{+\infty}\sum_{n=1}^{3}|i,p\rangle\langle i.p|\,dp = 1 . \qquad (5)$$

The dipole interaction is given by:

$$\hat{H}_{AL} = -\hat{\boldsymbol{\mu}}\cdot\hat{\boldsymbol{F}} , \qquad (6)$$

where the electric dipole momentum operator is:

$$\hat{\boldsymbol{\mu}} = e\,\hat{\boldsymbol{r}}_{\boldsymbol{e}} , \qquad (7)$$

where e is the charge of the electron and $\hat{\boldsymbol{r}}_{\boldsymbol{e}}$ is the position operator for the electron, and the total field operator is given by

$$\hat{F}(\hat{z},t) = \frac{1}{2}\left(\vec{e}_{32}F\,e^{-i\omega t + ik\hat{z}} + \vec{e}_{12}F\,e^{-i\omega t - ik\hat{z}}\right) +$$
$$\frac{1}{2}\left(\vec{e}_{32}F^*\,e^{i\omega t - ik\hat{z}} + \vec{e}_{12}F^*\,e^{i\omega t + ik\hat{z}}\right) , \qquad (8)$$

where $\vec{e}_{32}$ and $\vec{e}_{12}$ are the polarizations of the forward and backward laser fields. The dipole moment operator only affects the internal quantum numbers of the atom. In our system, the two laser fields are polarized in such a way that:

$$\langle 3|\hat{\boldsymbol{\mu}}\cdot\vec{e}_{32}|2\rangle = \langle 1|\hat{\boldsymbol{\mu}}\cdot\vec{e}_{12}|2\rangle \equiv \mu \qquad (9)$$

are the only nonzero matrix elements.

To find the matrix expansion of the dipole interaction, we make use of the identity:

$$e^{+ik\hat{z}}|i,p\rangle = |i,p+\hbar k\rangle . \qquad (10)$$

Equation 10 can be proved using the transformation between position and momentum space which can be found in many quantum mechanic textbooks: [3]

$$|i,p\rangle = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{+\infty} e^{i\frac{pz}{\hbar}}|i,z\rangle\,dz , \qquad (11)$$

Operating on both sides of Equation 11 with $e^{+ik\hat{z}}$ yields:

$$e^{+ik\hat{z}}|i,p\rangle = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{+\infty} e^{i\frac{(p+\hbar kz)}{\hbar}}|i,z\rangle\,dz = |i,p+\hbar k\rangle, \qquad (12)$$

Using the identity of Equation 10 and substituting Equation 8 into Equations 6 gives the matrix elements of the dipole interaction as:

$$\hat{H}_{AL}|1,p\rangle = -\hbar E e^{-i\omega t}|2,p-\hbar k\rangle - \hbar E^* e^{-i\omega t}|2,p+\hbar k\rangle$$
$$\hat{H}_{AL}|2,p\rangle = -\hbar E e^{-i\omega t}|3,p+\hbar k\rangle - \hbar E^* e^{-i\omega t}|3,p-\hbar k\rangle$$
$$-\hbar E e^{-i\omega t}|1,p-\hbar k\rangle - \hbar E^* e^{-i\omega t}|1,p+\hbar k\rangle \qquad (13)$$
$$\hat{H}_{AL}|3,p\rangle = -\hbar E e^{-i\omega t}|2,p+\hbar k\rangle - \hbar E^* e^{-i\omega t}|2,p-\hbar k\rangle ,$$

where $E = \mu F/2\hbar$ is called the Rabi frequency of the laser field. [5]

Expanding the wave function in terms of the slowly varying dynamical variable $c_i(p,t)$ gives:

$$|\psi\rangle = \int \left[\begin{array}{c} c_1(p,t)e^{-i\omega t}|1,p\rangle + c_2(p,t)|2,p\rangle \\ +c_3(p,t)e^{-i\omega t}|3,p\rangle \end{array} \right] dp \qquad (14)$$

The second term does not have a time dependence since it represents the ground state of the 'V' atom. Inserting Equation 14 into Equation 3 and ignoring terms proportional to $e^{\pm i2\omega t}$ (the so-called rotational wave approximation [5]) we find a closed set of equations:

$$\begin{aligned}
\frac{d}{dt}c_2(p,t) &= -i\frac{p^2}{2M\hbar}c_2(p,t) - iE^*c_1(p-\hbar k,t) \\
&\quad - iE^*c_3(p+\hbar k,t) \\
\frac{d}{dt}c_1(p-\hbar k,t) &= +i\left[\delta - \frac{(p-\hbar k)^2}{2M\hbar}\right]c_1(p-\hbar k,t) \\
&\quad + iEc_2(p,t) \qquad (15) \\
\frac{d}{dt}c_3(p+\hbar k,t) &= +i\left[\delta - \frac{(p+\hbar k)^2}{2M\hbar}\right]c_3(p+\hbar k,t) \\
&\quad + iEc_2(p,t)\ ,
\end{aligned}$$

where:

$$\delta = \omega - \Omega_{32} - \omega_r = \omega - \Omega_{12} - \omega_r$$

is the laser detuning with respect to the recoil frequency shift.

The momentum distribution of the atoms is associated with $c_2(p,t)c_2^*(p,t)$. To form a closed set of equations such as Equations 15 involving the atomic momentum distribution, we introduce the density matrix elements:

$$\begin{aligned}
\rho_{11}(p,t) &= c_1(p-\hbar k,t)c_1^*(p-\hbar k,t) \\
\rho_{22}(p,t) &= c_2(p,t)c_2^*(p,t) \\
\rho_{33}(p,t) &= c_3(p+\hbar k,t)c_3^*(p+\hbar k,t) \\
\rho_{21}(p,t) &= c_2(p,t)c_1^*(p-\hbar k,t) \qquad (16) \\
\rho_{32}(p,t) &= c_3(p+\hbar k,t)c_2^*(p,t) \\
\rho_{13}(p,t) &= c_1(p-\hbar k,t)c_3^*(p+\hbar k,t) \\
\rho_{ij}(p,t) &= \rho_{ji}^*(p,t) \quad i \neq j
\end{aligned}$$

If we take spontaneous emission into consideration, we find that the equations of motion for the density matrix elements are governed by Equations 17, known for historical reasons as the GOB: [2,5]

$$\begin{aligned}
\frac{d}{dt}\rho_{11}(p,t) &= -\Gamma\rho_{11}(p,t) + i\left[E\rho_{21}(p,t) - E^*\rho_{12}(p,t)\right] \\
\frac{d}{dt}\rho_{33}(p,t) &= -\Gamma\rho_{33}(p,t) + i\left[E\rho_{23}(p,t) - E^*\rho_{32}(p,t)\right] \\
\frac{d}{dt}\rho_{12}(p,t) &= -\left[0.5\Gamma - i\,\delta'_{12}(p)\right]\rho_{12}(p,t) + \\
&\quad iE\left[\rho_{22}(p,t) - \rho_{11}(p,t)\right] - iE\rho_{13}(p,t) \\
\frac{d}{dt}\rho_{32}(p,t) &= -\left[0.5\Gamma - i\,\delta'_{32}(p)\right]\rho_{32}(p,t) + \\
&\quad iE\left[\rho_{22}(p,t) - \rho_{33}(p,t)\right] - iE\rho_{31}(p,t) \\
\frac{d}{dt}\rho_{31}(p,t) &= -\left[\Gamma - i\,\delta'_{31}(p)\right]\rho_{31}(p,t) + \\
&\quad i\left[E\rho_{21}(p,t) - E^*\rho_{32}(p,t)\right] \\
\frac{d}{dt}\rho_{22}(p,t) &= \Gamma\int_{-\hbar k}^{+\hbar k} dq\,N(q)\,\rho_{11}(p+\hbar k+q,t)\ + \qquad (17) \\
&\quad \Gamma\int_{-\hbar k}^{+\hbar k} dq\,N(q)\,\rho_{33}(p-\hbar k+q,t)\ - \\
&\quad i\left[E\rho_{21}(p,t) - E^*\rho_{12}(p,t)\right] - i\left[E\rho_{23}(p,t) - E^*\rho_{32}(p,t)\right], \\
\frac{d}{dt}\rho_{21}(p,t) &= \frac{d}{dt}\rho_{12}^*(p,t) \\
\frac{d}{dt}\rho_{23}(p,t) &= \frac{d}{dt}\rho_{32}^*(p,t) \\
\frac{d}{dt}\rho_{13}(p,t) &= \frac{d}{dt}\rho_{31}^*(p,t)
\end{aligned}$$

where Γ is the decay rate of the excited state, $N(q)$ is the probability of an excited atom of momentum $p+q$, where $|q| \leq \hbar k$, becoming a ground state atom with momentum p and:

$$\begin{aligned}
\delta'_{12}(p) &= \delta + \frac{k}{M}p = \delta + 2\,\omega_r\frac{p}{\hbar k} \\
\delta'_{32}(p) &= \delta - \frac{k}{M}p = \delta - 2\,\omega_r\frac{p}{\hbar k} \qquad (18) \\
\delta'_{31}(p) &= \delta'_{32}(p) - \delta'_{12}(p)\ .
\end{aligned}$$

In Equation 17, the effect of the vacuum has been taken care of by terms proportional to the excited decay rate Γ. Due to the randomness of the spontaneous emission, $N(q)$ is:

$$N(q) = \frac{3}{8\hbar k}\left[1 + \left(\frac{q}{\hbar k}\right)^2\right], \qquad (19)$$

assuming that the emitted photon is circularly polarized. [2] The increase in the ground state population by this process is represented by the two integrations in Equation 17.

THE STEADY STATE SOLUTION

To obtain the steady state solution, all the time derivatives in Equation 17 are set to zero. This leads to 9 coupled equations:

$$0 = \Gamma\rho_{11}(p,t) - i\left[E\rho_{21}(p,t) - E^*\rho_{12}(p,t)\right] \qquad (20a)$$

$$0 = \Gamma\rho_{33}(p,t) - i\left[E\rho_{23}(p,t) - E^*\rho_{32}(p,t)\right] \qquad (20b)$$

$$\begin{aligned}
0 &= \left[0.5\Gamma - i\,\delta'_{12}(p)\right]\rho_{12}(p,t) - iE\left[\rho_{22}(p,t) - \rho_{11}(p,t)\right] \\
&\quad + iE\rho_{13}(p,t) \qquad (20c)
\end{aligned}$$

$$\begin{aligned}
0 &= \left[0.5\Gamma - i\,\delta'_{32}(p)\right]\rho_{32}(p,t) - iE\left[\rho_{22}(p,t) - \rho_{33}(p,t)\right] \\
&\quad + iE\rho_{31}(p,t) \qquad (20d)
\end{aligned}$$

$$0 = \left[\Gamma - i\,\delta'_{31}(p)\right]\rho_{31}(p,t) - i\left[E\,\rho_{21}(p,t) - E^{*}\,\rho_{32}(p,t)\right] \qquad (20e)$$

$$0 = \Gamma\int_{-\hbar k}^{+\hbar k} dq\, N(q)\,\rho_{11}(p+\hbar k+q,t)\; + \;\Gamma\int_{-\hbar k}^{+\hbar k} dq\, N(q)\,\rho_{33}(p-\hbar k+q,t) - i\left[E\rho_{21}(p,t) - E^{*}\rho_{12}(p,t)\right] - i\left[E\rho_{23}(p,t) - E^{*}\rho_{32}(p,t)\right] \qquad (20f)$$

$$0 = \left[0.5\Gamma + i\delta'_{12}(p)\right]\rho_{12}(p,t) + iE^{*}\left[\rho_{22}(p,t) - \rho_{11}(p,t)\right] - i\,E^{*}\,\rho_{31}(p,t) \qquad (20g)$$

$$0 = \left[0.5\Gamma + i\delta'_{32}(p)\right]\rho_{23}(p,t) + iE^{*}\left[\rho_{22}(p,t) - \rho_{33}(p,t)\right] - i\,E^{*}\,\rho_{13}(p,t) \qquad (20h)$$

$$0 = \left[\Gamma + i\delta'_{31}(p)\right]\rho_{13}(p,t) + i\left[E^{*}\,\rho_{12}(p,t) - E\,\rho_{23}(p,t)\right] \qquad (20i)$$

A method of dealing with coupled equations is to first use a forward substitution to reduce the coupled equations to one equation involving one unknown and then use a backward substitution to obtain all the unknowns in terms of the one equation with one unknown. We begin the substitution process by solving Equations 20*c*, 20*d*, 20*g* and 20*h* for $\rho_{12}(p)$, $\rho_{21}(p)$,$\rho_{32}(p)$ and $\rho_{23}(p)$ in terms of the populations $\rho_{31}(p)$ and $\rho_{13}(p)$. We then substitute these results back into Equations 20*a*, and 20*b*. From this, we find two sources of contributions to the excited populations. The first can be traced to the single-photon process.

The effect of the single photon process on the atomic populations can be represented by the single photon absorption (emission) rates:

$$a_{12}(p) = \frac{\Gamma I}{0.25\,\Gamma^2 + \delta'^{\,2}_{12}}$$
$$a_{21}(p) = \frac{\Gamma I}{0.25\,\Gamma^2 + \delta'^{\,2}_{32}} \qquad (21)$$

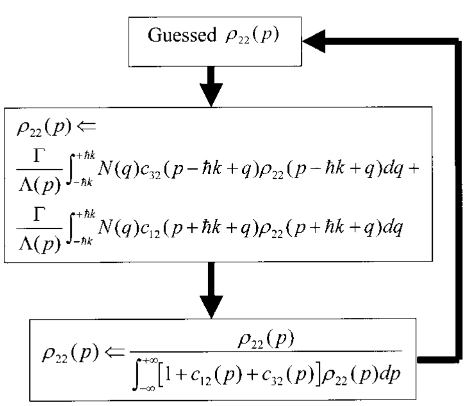

Figure 2
Flow chart of the numerical method used in the self-consistent approach.

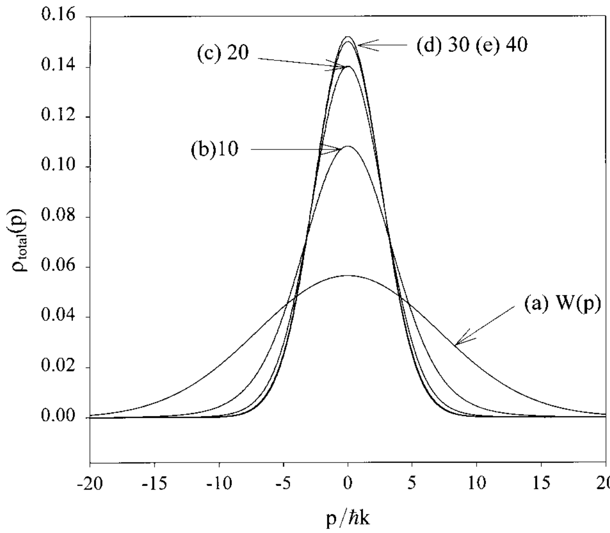

Figure 3
Total momentum distribution, $\rho_{total}(p)$ generated after b) 10, c) 20, d) 30 and e) 40 iterations. The initial guess is a Gaussian function with width $\Delta p_D = 10\hbar k$. Other parameters used in the simulations are: ω_r=0.02657Γ, E = 0.1Γ and δ = -0.6Γ

where $I = |E|^2$ is the intensity of the laser. The second contribution is due to the excited coherences $\rho_{31}(p)$ and $\rho_{13}(p)$ established by the two-photon process.[6] The two photon process is characterized by the net escaping rate of the atoms at level 1:

$$A_{11}(p) = 2I^2\,\frac{(0.25\,\Gamma_1^{\,2} + \delta'^{\,2}_{12})G_{31} + \Gamma_1\,\delta'_{12}\,S_{31}}{(0.25\,\Gamma_1^{\,2} + \delta'^{\,2}_{12})^2\,(G_{31}^{\,2} + S_{31}^{\,2})}, \qquad (22)$$

the net escaping rate of atoms at level 3:

$$A_{33}(p) = 2I^2\,\frac{(0.25\,\Gamma_3^{\,2} + \delta'^{\,2}_{32})G_{31} + \Gamma_3\,\delta'_{32}\,S_{31}}{(0.25\,\Gamma_3^{\,2} + \delta'^{\,2}_{32})^2\,(G_{31}^{\,2} + S_{31}^{\,2})}, \qquad (23)$$

and finally the population exchange rate between levels 1 and 3:

$$A(p) = 2I^2\,\frac{(0.25\,\Gamma_1^{\,2} + \delta'_{12}\delta'_{32})G_{31} + \Gamma_3\,(\delta'_{12} - \delta'_{23})\,S_{31}}{(0.25\,\Gamma_1^{\,2} + \delta'^{\,2}_{12})\,(0.25\,\Gamma_3^{\,2} + \delta'^{\,2}_{32})\,(G_{31}^{\,2} + S_{31}^{\,2})}, \qquad (24)$$

where:

$$G_{31}(p) = \Gamma_1 + \frac{0.5\,\Gamma_1\,I}{0.25\,\Gamma_1^{\,2} + \delta'^{\,2}_{12}} + \frac{0.5\,\Gamma_3\,I}{0.25\,\Gamma_3^{\,2} + \delta'^{\,2}_{23}}$$
$$S_{31}(p) = \delta'_{31} + \frac{I\,\delta'_{12}}{0.25\,\Gamma_1^{\,2} + \delta'^{\,2}_{12}} - \frac{I\,\delta'_{32}}{0.25\,\Gamma_3^{\,2} + \delta'^{\,2}_{23}}. \qquad (25)$$

These rates are deduced by expressing $\rho_{31}(p)$ and $\rho_{13}(p)$ in terms of the population terms with the help of Equations 20*c*, 20*d*, 20*e*, 20*g*, 20*h* and 20*i*. These steps reduce Equations 20*a* and 20*b* into steady state rate equations from which the excited state populations, $\rho_{11}(p)$ and $\rho_{33}(p)$ can be written in terms of the ground state population, $\rho_{22}(p)$ as:

$$\rho_{11}(p) = c_{12}(p)\,\rho_{22}(p)$$
$$\rho_{33}(p) = c_{32}(p)\,\rho_{22}(p)\,. \qquad (26)$$

where

$$c_{12}(p) = \frac{R_{12}(\Gamma_3 + R_{32}) - A\,[\Gamma_3 + A]}{(\Gamma_1 + R_{12})\,(\Gamma_3 + R_{32}) - A^2}$$
$$c_{32}(p) = \frac{R_{32}(\Gamma_1 + R_{12}) - A\,[\Gamma_1 + A]}{(\Gamma_1 + R_{12})\,(\Gamma_3 + R_{32}) - A^2}\,, \qquad (27)$$

and

$$R_{12}(p) = a_{12}(p) - A_{11}(p)$$
$$R_{32}(p) = a_{32}(p) - A_{33}(p) \qquad (28)$$

Finally, we combine Equations 20*a*, 20*b* and 20*f* and transform the results with the help of Equation 26 into an integral equation the single unknown function, $\rho_{22}(p)$:

$$\rho_{22}(p) = \frac{\Gamma}{\Lambda(p)}\int_{-\hbar k}^{+\hbar k} dq\, N(q)\, c_{32}(p - \hbar k + q)\, \rho_{22}(p - \hbar k + q)$$
$$+ \frac{\Gamma}{\Lambda(p)}\int_{-\hbar k}^{+\hbar k} dq\, N(q)\, c_{12}(p + \hbar k + q)\, \rho_{22}(p + \hbar k + q), \quad (29)$$

where

$$\Lambda(p) = \Gamma_1\, c_{12}(p) + \Gamma_3\, c_{32}(p)\,. \qquad (30)$$

In addition, $\rho_{22}(p)$ is constrained by the normalization condition:

$$\int_{-\infty}^{+\infty}\Big[\rho_{11}(p) + \rho_{22}(p) + \rho_{33}(p)\Big]\,dp = 1\,, \qquad (31)$$

which can be written using Equation 20 as:

$$\int_{-\infty}^{+\infty}\Big[1 + c_{12}(p) + c_{3c}(p)\Big]\,\rho_{22}(p)\,dp = 1\,. \qquad (32)$$

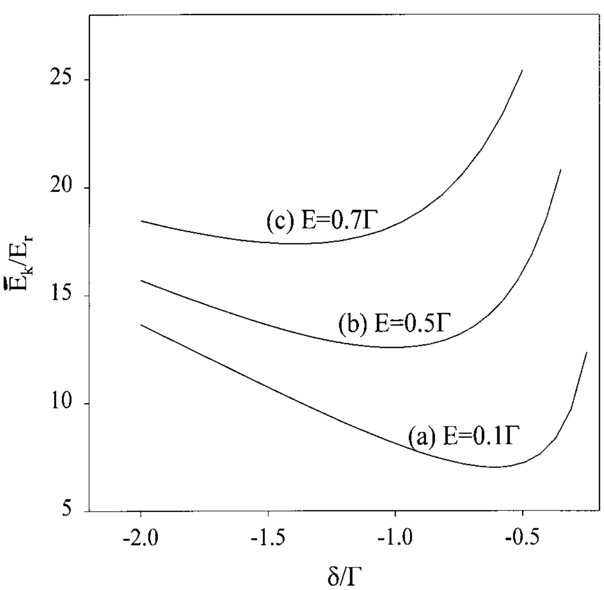

Figure 4
The average kinetic energy vs laser detuning for a) E = 0.1Γ, b) E = 0.5Γ and c) E = 0.5Γ. ω_r for these curves is 0.02657Γ

NUMERICAL SIMULATION AND DISCUSSION

The key determining the steady-state atomic distribution is the solution of the integral Equation 29. Once $\rho_{22}(p)$ is determined, the momentum distributions in the excited states can be obtained using Equations 26. We take advantage of the normalization condition in Equation 32 and propose a method based on the concept of self-consistency to solve Equation 29.

The self-consistent loop is illustrated in Figure 2. A distribution function is guessed for $\rho_{22}(p)$. With the guessed value for $\rho_{22}(p)$, the right side of Equation 29 is evaluated explicitly to yield a new momentum distribution for the ground state. This newly produced $\rho_{22}(p)$, after a normalization according to Equation 32, assumes the role of the guessed one and the process is repeated. When the shape of the function $\rho_{22}(p)$ does not change with further iterations, the state of self-consistency is reached and the unchanged $\rho_{22}(p)$ is the desired steady state solution. If we do not reach a shape for $\rho_{22}(p)$ that does not change, the steady state solution does not exist.

In our simulation, the unit of momentum is chosen to be $\hbar k$ and Γ is the unit for any rates and frequencies. The guessed value for $\rho_{22}(p)$ is assumed to be a normalized Gaussian function:

$$\rho_{22}(p) = \frac{1}{\sqrt{2\pi}\,\Delta p_D}\, e^{-\left(\frac{p}{\Delta p_D}\right)^2}, \qquad (33)$$

where Δp_D is the half width of the distribution. The momentum is sampled at a rate between 10 and 20 divisions per $\hbar k$. In the simulation, we calculate the total momentum distribution, defined as:

$$\rho_{total}(p) = c_1(p)\,c_1^*(p) + c_2(p)\,c_2^*(p) + c_3(p)\,c_3^*(p)\,. \qquad (34)$$

Using Equation 16, Equation 34 can be written as:

$$\rho_{total}(p) = \rho_{11}(p + \hbar k) + \rho_{22}(p) + \rho_{33}(p - \hbar k) \qquad (35)$$

We also calculate the average kinetic energy:

$$\bar{E}_k = \langle\psi|\frac{\hat{p}^2}{2M}|\psi\rangle = \int_{-\infty}^{+\infty}\frac{p^2}{2M}\,\rho_{total}(p)\,dp\,, \qquad (36)$$

which is a direct measure of the temperature of the atomic ensemble. The integrations are all done using Simpson's rule. [7]

We first consider the λ = 1.083 μm transition in ^{4}He atoms for which $\omega_r = 0.02657\Gamma$. Figure 3 shows several results for low intensity $E = 0.1\Gamma$ and $\delta = -0.6\Gamma$. We start with the guessed ρ_{22}(p) with $\Delta p_D = 10\hbar k$ (curve *a*) and then did 10 iterations (curve *b*), 20 iterations (curve *c*), 30 iterations (curve *d*) and 40 iterations (curve *e*). The difference between the results for 30 iterations and 40 iterations is extremely small, so we conclude that the state of self-consistency is reached in about 40 iterations. The width of the momentum distribution is $3\hbar k$, and the average kinetic energy is 7.04 E_r.

Figure 4 shows how the average kinetic energy changes with the laser detuning δ for different values of E. The average kinetic energy reaches a minimum, $\bar{E}_{k\,\min}$, at laser detuning δ_{min}. Note that both $|\delta_{min}|$ and $\bar{E}_{k\,\min}$ increase with the Rabi frequency.

The existence of an optimal detuning at which the kinetic energy is minimum (meaning the lowest temperature) is a manifestation of laser cooling. If the detuning is too far below the atomic transition, the net force resulting from the Doppler shift between the counter and copropagating laser fields is relatively weak, so the cooling efficiency is not high, If, on the other hand, the detuning is too close to the atomic transition, atoms of even moderate velocities "see" the frequencies of both lasers to be larger than the atomic transition frequency, so the counter propagating laser is further away in energy from the atomic transitions than the copropagating one. The net effect is that the motion of these atoms, instead of being slowed down, is accelerated, leading to heating of the atoms.

In fact, we found that if δ is some frequency above δ_{min}, the momentum distribution keeps expanding as the number of iterations increases. Physically this means that momentum diffusion due to the spontaneous emission overcomes the radiation cooling, so a steady state distribution cannot be reached.

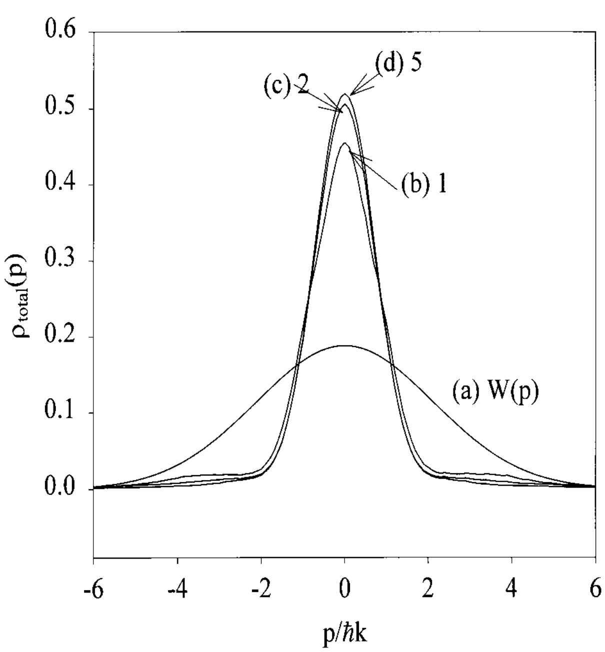

Figure 5
Total momentum distribution, $\rho_{total}(p)$ generated after b) 1, c) 2, d) 5 iterations. The initial guess is a Gaussian function with width $\Delta p_D = 3\hbar k$. Other parameters used in the simulations are: ω_r=10Γ, E = 10Γ and δ = -38Γ.

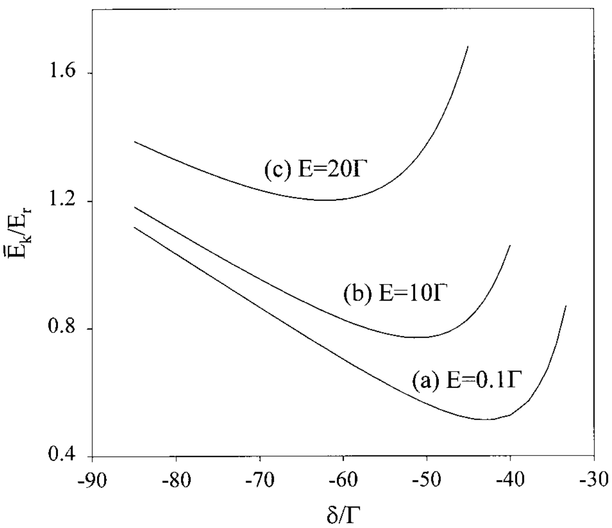

Figure 6
The average kinetic energy vs laser detuning for a) E = 0.1Γ, b) E = 10Γ and c) E = 20Γ. ω_r for these curves is 10Γ.

We worked out a case where the recoil shift was much larger than the excited decay rate: $\omega_r = 10\Gamma$, $E = 10\Gamma$ and $\delta = -38\Gamma$. We began with an initial Gaussian function with $\Delta p_D = 3\hbar k$. Figure 5 shows the results. Curve *a* is the initial shape of $\rho_{22}(p)$. Curve *b* is after 1 iteration, curve *c* after 2 iterations and curve *d* is after 5 iterations. Remarkably, the state of self-consistency is reached after only several iterations. The width of the momentum after these iterations is about $0.9\hbar k$, much narrower than that shown in Figure 3. As ω_r/Γ increases, the momentum distribution narrows. The average kinetic energy was 1.43 E_r.

In both of the cases discussed, the momentum width is on the order of the single photon momentum. As a result, a momentum change of $\hbar k$ is not negligible. The full quantum mechanical treatment of the atomic variables is justified.

The change in the average kinetic energy vs the laser detuning is displayed in Figure 6 for different Rabi frequencies. Again, it shows that there exists an optimal laser detuning at which the average kinetic energy, or equivalently the temperature of the atoms, is a minimum.

Previous theoretical work [2] predicted that for narrow momentum distributions (large ω_r/Γ) in the low intensity limit (small E/Γ), $\delta_{min} \approx -4.5\omega_r$ and $\bar{E}_{k\,\min} \approx 0.530\ \hbar\ \omega_r$. The curve in Figure 6 is produced with parameters close to these limits (ω_r/Γ=10>>1 and E/Γ = 0.1 << 1). The minimum kinetic energy is reached at $\delta_{min} \approx -43.1\Gamma = -4.31\omega_r$ with $\bar{E}_{k\,\min} \approx 0.513\ \hbar\ \omega_r$. These results are indeed quite close to the theoretical predictions.

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